UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 54 JANUARY 1912

MECHANICAL STRESSES IN TRANSMISSION LINES

BY A. GUELL, RESEARCH FELLOW, ELECTRICAL ENGINEERING DEPARTMENT

CONTENTS

I. INTRODUCTION

PAGE

1. Preliminary 3
2. Object of Bulletin 3
3. Acknowledgments 3
4. Notation 4

II. STRESSES DUE TO DEAD WEIGHT

5. Classification 4
6. Weight of Snow or Ice Coating 5
7. Weight of Foundation 5

III. STRESSES DUE TO WIND PRESSURE

8. Lateral Stresses 10
9. Wind Pressure on Flat Surfaces 10
10. Wind Pressure on Oblique Surfaces 12
11. Wind Pressure on Large and Small Surfaces 12

IV. TENSION IN AERIAL LINES

12. Analytical Discussion 14
13. Approximate or Parabola Method 21

V. PRACTICAL USE OF FOREGOING FORMULAS

VI. CATENARY METHOD

Digitized by the Internet Archive
in 2024

https://archive.org/details/university-of-illinois-engineering-experiment-bulletin_1912-01_54

MECHANICAL STRESSES IN TRANSMISSION LINES

I. Introduction

1. *Preliminary.*—Owing to the fact that the mechanical stresses in transmission lines vary within unusually wide limits, many electrical engineers have overlooked the fact that otherwise well-designed and expensive electric power stations have been seriously crippled because the transmission line, though electrically strong, was mechanically weak. There are instances where it would be a commercial impossibility to design a transmission line that would withstand the local combined maximum wind pressure, ice load, and temperature range. If a telegraph or telephone line should be overturned by an exceptionally severe storm, accompanied by sleet on the wires, comparatively little inconvenience would result, owing to the fact that these small wires, though frequently numerous, carry harmless currents. The percentage of modern telegraph and telephone lines yearly destroyed is fairly small; and, for this reason, to build all the lines strong enough to resist the most exceptional wind pressure would be more expensive than to replace the few lines destroyed. Let a high-tension transmission line be overturned, however, and more serious results will follow. The transmission line is often the most expensive part of an electric power development; therefore, it should be built strong enough to resist the exceptionally violent local storms, and no expense should be spared to insure safety to life.

2. *Object of Bulletin.*—Transmission lines are, in general, subjected to stresses due to the following causes: (1) dead weight; (2) wind pressure; (3) changes of temperature; (4) tensional unbalancing. It is the object of this bulletin to discuss the first three of the above stresses.

3. *Acknowledgments.*—Acknowledgment is hereby made to Messrs. A. J. Bowie, Jr., (Electrical World, Vol. 48) and to J. Mayer

(Engineering News, Vol. 35) for the use of material from articles which appeared in the above mentioned magazines.

4. *Notation.*—The following notation will be used.

A = cross-sectional area of wire.
α = coefficient of linear expansion per degree F.
c = tension constant at lower temperature.
c_1 = tension constant at higher temperature.
E = modulus of elasticity of wire.
e = base of natural logarithms.
f = deflection at lower temperature.
f_1 = deflection at higher temperature.
ϕ = angle.
θ = angle.
H = tension at lowest point of wire (lower temperature).
H_1 = tension at lowest point of wire (higher temperature).
l = length of span.
L = length of wire at lower temperature.
L_1 = length of wire at higher temperature.
M,N = constants.
s = half length of catenary at lower temperature.
s_1 = half length of catenary at higher temperature.
T = tension at insulator at lower temperature.
T_1 = tension at insulator at higher temperature.
τ = temperature range.
u = constant.
w = resultant weight at lower temperature.
w_1 = resultant weight at higher temperature.
x = half length of span.
y = any ordinate; when maximum = f.

II. Stresses Due to Dead Weight

5. *Classification of Dead Weight Stresses.*—The dead weight consists of: (1) weight of poles or towers; (2) weight of one span of wire; (3) weight of snow or ice-coating, if any; (4) weight of foundation, if any.

Stresses (1) and (4) are, as a rule, easily taken care of by the bearing soil or sub-foundation, as they act vertically downwards, or nearly so. Stresses (2) and (3) have both a vertical and a horizontal component, but the horizontal component is the more important. Under

the heading, The Tension of Aerial Lines, it will be shown how great this horizontal component may become in extra long spans.

6. *Weight of Snow or Ice Coating.*—Snow or ice load becomes the more important, the smaller the diameter of the wire. This may easily be seen by considering two wires of the same length, but of different diameters. Assuming a coat of ice ½ in. thick, the increase in weight per unit length, on a wire 0.102 in. in diameter (10 B & S gauge) would be

$$\frac{\pi i}{4}[(1+0.102)^2-(0.102)^2]=1.204\ \frac{\pi i}{4}\ \ldots\ldots(1)$$

i = weight of ice in pounds per cubic inch

The increase in weight on a wire 0.409 in. in diameter (3-O B & S) would be

$$\frac{\pi i}{4}[(1+0.409)^2-(0.409)^2]=1.818\frac{\pi i}{4}\ldots\ldots\ldots(2)$$

The ratio of diameters is 1:4, but the ratio of increased load is 1:1.5, as may be seen from equations 1 and 2.

7. *Weight of Foundation.*—Any foundation may be regarded as having two components: (1) the bearing soil or sub-foundation; (2) the foundation proper, consisting of the materials forming a solid base for the superstructure. As geological conditions vary widely in different localities, an approximate knowledge of the characteristics of the soil under consideration is of importance.

(a) Bearing Soil.—Sometimes the bearing power of the soil must necessarily be increased. This can be accomplished by means of piles. Trautwine gives the following formula for determining the maximum load that a pile will stand:

$$W=\frac{0.023\sqrt[3]{H}\,w}{h+1}$$

where

W = ultimate load in tons of 2000 pounds;
H = fall in feet;
w = weight of hammer in pounds;
h = last sinking in inches.

This formula must be used with a factor of safety of from 1 to 10, depending upon local conditions.

Table 1 shows the safe bearing values of ordinary soils.

TABLE 1

SAFE BEARING VALUES OF SOILS

Kind of Material	Bearing Values tons per sq. ft.	
	Maximum	Minimum
Rock, the hardest, in thick layers in native bed		200
Rock, the softest, easily worn by water or exposure to the weather		18
Clay, in thick beds, always dry	6	4
Clay, in thick beds, moderately dry	4	2
Clay, soft beds	2	1
Gravel and coarse sand, well cemented	10	8
Sand, compact and well cemented	6	4
Sand, clean and dry	4	2
Quicksand and alluvial soils	1	0.5

Table 2, compiled from various authorities, gives the crushing strengths and moduli of rupture of the principal materials used for foundations.

TABLE 2

CRUSHING STRENGTHS AND MODULI OF RUPTURE

Kind of Material	Moduli of Rupture lb. per sq. in.	Crushing Strengths lb. per sq. in.
Granite	1800	10400
Limestone, common varieties	1500	8670
Oolitic limestone	2338	6700
Sandstone (brownstone)	2160	8300
Concrete, 1 month, 1 part portland cement, 2 parts sand, and 4 parts broken stone	250	1500
Brick laid in portland cement, 1 to 2 mortar		2900
Brick laid in Rosedale cement, 1 to 2 mortar		1700

(b) *The Foundation.*—Stresses in foundations are due to two causes: (1) dead weight of poles or towers, plus weight of one span of wire, together with any snow or ice coating: (2) stresses due to wind pressure; tensional unbalancing caused by spans of different lengths, breaking of one, two, or all of the wires, changes in the direction of the line, changes of level, etc.

The total dead weight divided by the area of the foundation, plus the unit load produced by the maximum wind pressure, (see Stresses Due to Wind Pressure), plus a factor representing the unit load produced by tensional unbalancing, are represented by the following formula:

$$L = \frac{W}{A} + \frac{Ml}{2I} + U \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where

L = load on sub-foundation per unit area;
W = total dead weight;
A = area of foundation;
M = moment of the wind;
I = moment of inertia of the area of the base about a gravity axis perpendicular to the direction of wind;
l = distance of most remote point of base from this axis;
U = factor representing unit load produced by tensional unbalancing.

Considering a common case, in which the towers rest on square bases, the direction of the wind is at right angles to the line, and the line is straight, there is obtained for the maximum intensity of pressure on the footing:

$$L = \frac{W}{A} + \frac{l}{2I'}\sqrt{M^2 + N^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which:

l = length of one side of square base;
I' = moment of inertia of base about a gravity axis parallel to one side;
N = moment due to tensional unbalancing.

and the other symbols have the same meaning as above.

Sometimes it is found necessary to extend the base of a foundation in order not to overload the bearing soil. To show how to determine the safe projection of this base, a simple case will be considered. (Fig. 1).

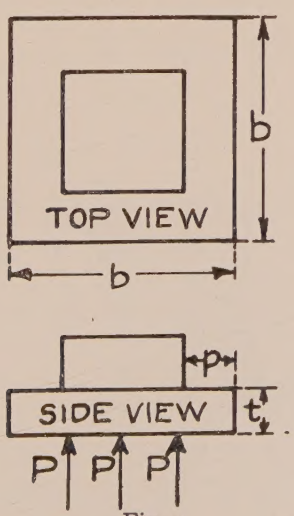

Fig. 1

Let P = the pressure in lb. per sq. in. at the bottom of the foundation;
R = the modulus of rupture of the material of the foundation in lb. per sq. in;
p = the projection in inches;
t = the thickness of the base in inches;
b = the breadth of the base in inches;

The projecting part, p, of the base may be considered as a cantilever beam uniformly loaded. Therefore, the bending moment of this beam may be represented by the formula

$$Ppb\frac{p}{2} = \frac{I}{6} Rbt^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Simplifying

$$p = t\sqrt{\frac{R}{3P}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

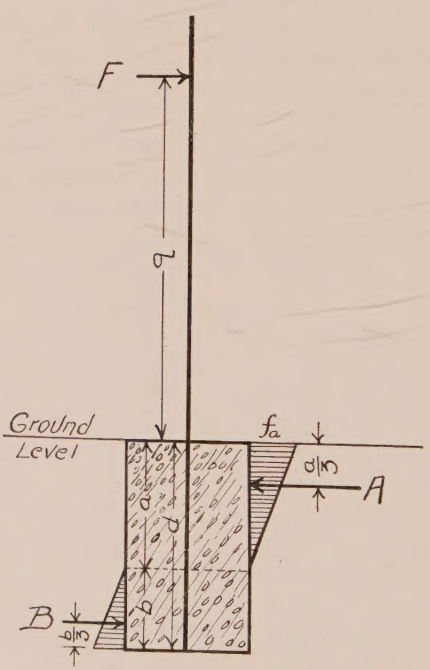

Fig. 2

A factor of safety ranging from 5 to 10 should be used in connection with this formula (6).

Fig. 1 may also be taken as representing one of the four separate masses of concrete needed, when the four posts of a tower are placed so far apart that it would be uneconomical to use a single concrete block to support the whole structure. It could also represent the slightly different case (on account of the earth on top) when the four blocks rest on a platform usually made of reinforced concrete.

The stresses in a foundation and the accompanying lateral earth pressures which are set up when the tower is subjected to lateral pressures will now be considered. Fig. 2 represents a longitudinal section of a prismatic concrete block of depth d which supports a tower which is subjected to lateral pressures. The longitudinal section of the block is taken in a plane parallel to the resultant lateral pressure F which acts at a distance q above the ground level. Neglecting friction of ground against the sides of the concrete block, the overturning moment of F is resisted by earth pressures having resultants A and B on opposite sides of the block. The intensity of earth pressure is assumed to vary in a straight line relation from zero at some section along the block to a maximum in one direction at the ground level (f_a), and to a maximum in the other direction at the bottom of the block. If the block is of uniform cross-section, the point of application of A will be one-third the distance from the ground level to the section of zero lateral pressure; similarly, the point of application of B will be one-third the distance from the bottom of the block to the section of zero lateral pressure. Using the notation of Fig. 2, we obtain the following equations for equilibrium. Taking moments about B,

$$F\left(q + d - \frac{b}{3}\right) = \frac{2Ad}{3},$$

$$B + F = A$$

also

$$\frac{A}{B} = \frac{a}{b}$$

and

$$a + b = d$$

From the above equations,

$$A = \tfrac{7}{8}F + \tfrac{3}{4}\frac{Fq}{d} \pm \frac{F}{8}\sqrt{17 + 36\frac{q}{d} + 36\frac{q^2}{d^2}}$$

and

$$a = \frac{Ad}{2A - F}$$

From the straight line relation of the lateral earth pressures,

$$f_a = \frac{2A}{a\,w},$$

in which w is the width of the block perpendicular to the line of action of F. f_a should not exceed the safe bearing pressure for the soil around the block. The maximum bending moment on the block is

$$\frac{2Bd}{3} = \frac{2(A—F)d}{3}$$

and the maximum fiber stress due to bending equals this maximum moment divided by the section modulus of a cross-section of the block about an axis perpendicular to the line of action of F. This fiber stress should not be greater than about one-sixth the strength of the material of the block in cross-bending.

III. Stresses Due to Wind Pressure

8. *Lateral Stresses.*—The chief lateral stresses to which transmission line poles or towers are subjected are due to wind pressure acting on the wires, and on the poles or towers themselves; (the pressure on cross-arms and insulators may ususally be neglected).

The usual assumed wind pressures in England, Germany, Switzerland and the United States are: 56, 26, 21, and 30 lb. per sq. ft. of exposed area. These pressures are for wind acting normally to a flat surface. The effective area offered by a cylinder to the pressure of wind is theoretically two-thirds of the diameter multiplied by the length. In Switzerland, 0.7 of the diameter times the length is allowed; in England, 0.6.

9. *Wind Pressure on Flat Surfaces.*—Air pressure, especially on curved surfaces, is a very complex quantity, and, so far as the writer is aware, no exact mathematical treatment of the subject has been given. This is due to the fact that when a body immersed in a fluid is moving in the latter (or vice versa), there will generally exist, at every point of the surface of the body, a pressure different from that which would have existed without this movement. When there is not such motion, the resultant of all the elementary forces due to the static pressure of this fluid is equal to zero. If the body is in motion, this can easily be understood, remembering that the pressure at any point can be regarded as composed of the pressure due to the direct stream of air flowing normal to the surface in question, and the reflected air streams from other points of the surface near by.

Drs. Finzi and Soldati* give an account of a most interesting series of experiments which they performed in determining air pressures. The series of tests includes the determination of the normal pressure on planes, cylinders, spheres, and bodies of other shapes. The method of determination consisted in the use of a tube connected to

*Engineering (London) March 17, April 14, 1905.

small holes, normal to the surfaces, the other end of the tube being connected to a pressure gauge. Thus the pressure over the entire surface could be observed by point measurements, the pressure at any point being of course normal to the surface.

Previous experiments on wind pressure had been made by measuring the pressure as a whole (dynamometric method) without making any determination as to the distribution over the various regions of the body, and this very fact explains why eminent authorities obtained radically different results. But the experiments of Drs. Finzi and Soldati have proved conclusively that the wind pressure on any surface was not evenly distributed, and that, in all cases, the pressures vary directly with the first power of the velocity heads. In the consideration of flat surfaces normal to the wind, the pressure may be regarded as composed of two parts: (1) front pressure, (2) back pressure. The front pressure is greatest at the center of the figure, where its highest value is equal to that due to the velocity head. It falls off towards the edges. The gross front pressure for a circle is 75 per cent of that due to the velocity head; for a square, it is 70 per cent., and for a rectangle whose length is very long compared with its width it is 83.3 per cent.

Back pressures are nearly uniform over the whole area except at the edges. In the above-mentioned experiments, the striking fact is brought out that the back pressure is dependent on the perimeter of the plane, and will vary between negative values of 40 and 100 per cent of the velocity head, its value being expressed by the equation,

$$P = \frac{V^2 W}{2g}(0.4 + 0.15l - 0.0044\ l^2)$$

where $\frac{V^2 W}{2g}$ is the velocity head and l is the perimeter in meters. This equation holds up to $l = 4$. From what precedes, the maximum total pressure on an indefinitely long rectangle of measurable width is 1.83 times the velocity head pressure. For a very small square, the coefficient may be as small as 1.1. According to the theory usually accepted, this coefficient should be 2.

Air pressures per unit area may be obtained by means of the following formula:

$$p = K\frac{V^2 W}{2g}$$

where

V = velocity of wind per second;
W = weight of air per unit cube;

g = acceleration of gravity in corresponding units.

Table 3, calculated from the above-mentioned article on air pressures, gives the coefficient K for various surfaces.

10. *Wind Pressure on Oblique Surfaces.*—The front-face pressure at an angle of less than 90° can be determined by means of the following formula:

$$p = \frac{V^2 W \sin^{4/3} \alpha}{2g}$$

TABLE 3

Description of Surface	Front	Back	K
10 cm. diameter disc	0.750	0.447	1.197
30 cm. diameter disc	0.754	0.538	1.292
100 cm. diameter disc	0.747	0.813	1.560
10 x 10 cm. square	0.697	0.453	1.150
30 x 30 cm. square	0.696	0.572	1.268
100 x 100 cm. square	0.697	0.893	1.690
10 x 30 rectangle	0.752	0.516	1.268
10 x 100 rectangle	0.771	0.713	1.484
30 x 100 rectangle	0.753	0.757	1.510
4.8 mm. diameter x 25 cm. cylinder	0.222	0.401	0.626
4.8 mm. x 25 cm. rectangle	0.756	0.488	1.244
Long rectangle	0.833	1.000	1.833

Experiments carried on by Drs. Finzi and Soldati proved that the exponent varies between 1.22 and 1.42. The back depression for angles, comprised between 60° and 90°, is fairly uniform over the whole area, and very similar to the values at 90° given in Table 3.

11. *Wind Pressure on Large and Small Surfaces.*—There is a great difference between wind pressure on large and small surfaces. This arises from the fact that the wind pressure acts in gusts or is oscillating, and the oscillations are not synchronous over any considerable extent of space. The English standard for wind pressure is 56 lb. per sq. ft. of plane exposed surface, 60 per cent of this being for cylindrical surfaces (counting diameter into length as exposed area). In this country, 50 lb. per sq. ft. is a frequent standard; but most American engineers are of the opinion that such pressures occur only over small surfaces, and that 30 lb. per sq. ft. of exposed surface is a liberal allowance for areas extending over considerable distances. The New York building code and many other building codes prescribe for high buildings 30 lb. per sq. ft.

The observed wind pressure greatly decreases as one approaches the surface of the ground. This is an additional reason why an assumed

wind pressure of more than 30 lb. per sq. ft. is not warranted for a transmission line, the towers of which are rarely more than 60 ft. high. For the cylindrical parts of a transmission line (poles, wires, etc.), as pressures on cylindrical surfaces have been shown by experiment to be from 50 to 60 per cent of those on plane surfaces, a pressure of 18 lb. per sq. ft. (counting diameter into length as area) is, therefore, a liberal assumption.

Mr. H. W. Buck* has given the results of a series of wind-pressure experiments made at Niagara on a 950-ft. span of 0.58-in. stranded cable, erected so as to be normal to the usual wind. From the data obtained, the following formula was derived:

$$p = 0.0025\ V^2$$

where

p = pressure in lb. per sq. ft. of projected area;
V = wind velocity in miles per hour.

For solid wire, previous experimenters had derived the formula:

$$p = 0.002\ V^2$$

Mr. Buck attributes the 25 per cent increase to the fact that, for a given diameter, cable presents increased surface for wind pressure. Unfortunately he mentions neither the barometer nor the temperature. Fig. 3 was drawn, using Mr. Buck's formula. He points out in his

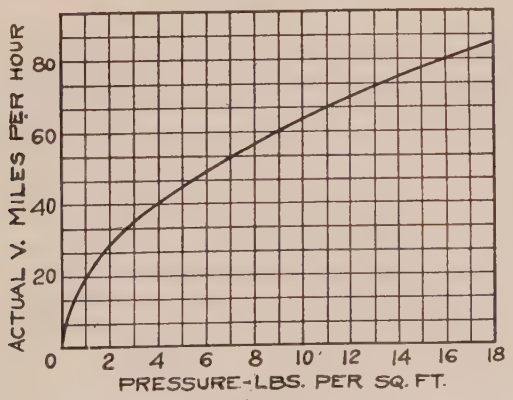

Fig. 3

paper that in the reports of the U. S. Weather Bureau, the wind velocities are higher than the actual velocities on a level with the average transmission line. This, of course, is due to the fact that

*Paper read at the World's Fair in St. Louis, 1904.

the recording instruments are usually located on high, exposed places. Fig. 4 shows graphically his tabular comparison of indicated and actual velocities.

It is highly desirable that proper investigations should be conducted along the line of suitable coefficients for wind-pressure, to see the effect, if any, which the wire diameter exerts on the pressure for a given projected area, and to determine whether the values should not be still greater for cable than for wire.

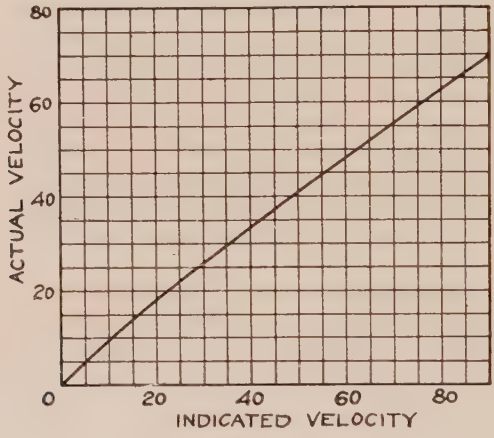

Fig. 4

Based on the fact that the tangent of the angle made by the plane of the wire with the vertical shows the ratio of the wind-pressure to the weight of the wire, A. J. Bowie, Jr., has suggested the following simple and feasible method for keeping record of or measuring the wind pressure on wires. A pivoted lever, provided with a pencil at the upper end, rests constantly against the wire at the center of the span. The pencil traces a diagram on paper from which can be determined the lateral sway of the wire and hence the wind pressure.

IV. Tension in Aerial Lines.*

12. *Analytical Discussion.*—Let ABC represent a wire fastened to insulators A and C, or in other words, imagine ABC to be any span of a transmission line. The internal force or tension in a perfectly flexible string, if hung as shown in Fig 5, would be directed

*The analytical discussion immediately following is substantially that given in text-books on Theoretical Mechanics. It is inserted here for the sake of completeness in the presentation.

at every point along the tangent line to the string; but this is not exactly true of a wire; for, if an attempt be made to bend an imperfectly flexible string, or a conductor, a certain amount of resistance proportional to the degree of rigidity of the string or wire is encountered. However, as the wire under consideration hangs freely, gravity being the only force acting upon it, for all practical purposes, the curve ABC may be considered as a catenary. This curve would be a perfect catenary if the wire were inextensible, perfectly flexible, and of uniform cross-section and density.

The properties of the curve ABC will be the same whether or not the points A and C are at the same level; but, for the sake of simplicity, the equation of this curve, assuming A and C to be in the same horizontal line, will be deduced.

Consider a particle P of the wire (Fig. 5). This particle, as

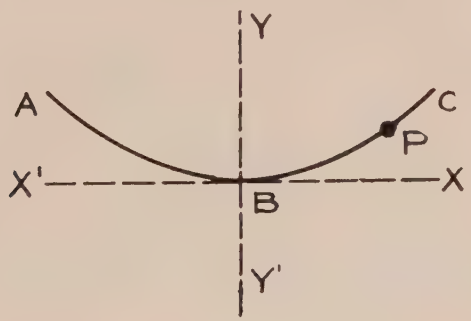

Fig. 5

shown in Fig. 6, is kept in equilibrium by the tensions T_1 and T_2, and by its weight, W acting downwards. Resolving these forces horizontally,

$$T_1 \cos \theta_1 - T_2 \cos \theta_2 = 0$$
$$T_1 \cos \theta_1 = T_2 \cos \theta_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

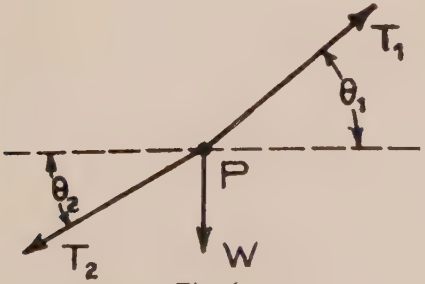

Fig. 6

That is, the horizontal components of the tensions in the different portions of the wire are the same.

The equation of the curve shown in Fig. 5 will now be deduced. The X-axis will be located at B, the lowest point of the catenary and the Y-axis will also pass through this point. Fig. 7 shows as free and in equilibrium, the right-hand half of the catenary ABC (Fig. 5). The tensions H and T, and the weight W, of the wire make this equilibrium stable. For algebraic convenience, make

$$H = cw \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

That is, the tension H at the lowest point of the curve may be considered replaced by the weight of a length c of wire weighing w pounds

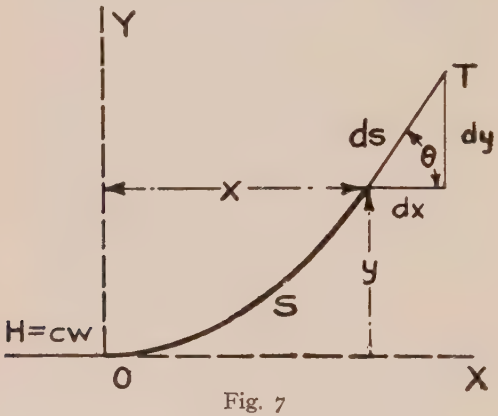

Fig. 7

per unit length. Calling s the length of the curve shown in Fig. 7, resolving vertically,

$$T\frac{dy}{ds} = sw = W \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Resolving horizontally

$$T\frac{dx}{ds} = cw = H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Dividing (3) by (4) and squaring

$$\frac{dy^2}{dx^2} = \frac{s^2}{c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Remembering that

$$dy^2 = ds^2 - dx^2,$$

substituting in (5), and solving for dx,

$$dx = \frac{c\,d\,s}{\sqrt{c^2 + s^2}}$$

and

$$x = c \int_0^s \frac{ds}{\sqrt{c^2 + s^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Equation (6) can be integrated as follows:

$$\int_0^s \frac{ds}{\sqrt{c^2 + s^2}} = \int_0^s \frac{ds}{\sqrt{c^2 + s^2}} \times \frac{s + \sqrt{c^2 + s^2}}{s + \sqrt{c^2 + s^2}} =$$

$$\int_0^s \frac{\sqrt{c^2 + s^2}\ ds + s\,d\,s}{\sqrt{c^2 + s^2}\ \left[s + \sqrt{c^2 + s^2}\right]} =$$

$$\int_0^s \frac{ds + \dfrac{s\,d\,s}{\sqrt{c^2 + s^2}}}{s + \sqrt{c^2 + s^2}} = \left[\log_e \left(s + \sqrt{c^2 + s^2}\right)\right]_0^s$$

$$= \log_e \frac{s + \sqrt{c^2 + s^2}}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Substituting (7) in (6)

$$x = c \log_e \frac{s + \sqrt{c^2 + s^2}}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Remembering that

$$dx^2 = ds^2 - dy^2,$$

substituting in (5), and solving for y,

$$y = \int_0^s \frac{s\,d\,s}{\sqrt{c^2 + s^2}} = \left[\sqrt{c^2 + s^2}\right]_0^s$$

Therefore

$$y = \sqrt{c^2 + s^2} - c \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Solving for c,

$$c = \frac{s^2 - y^2}{2y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Fig. 8 illustrates a principle which will be found of great use

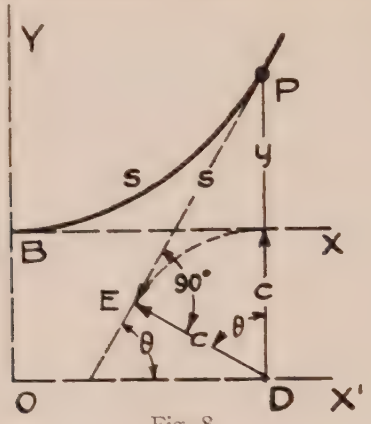

Fig. 8

when it is necessary to draw an accurate catenary.
From equation (3) and (4)

$$\frac{T \sin \theta}{T \cos \theta} = \frac{sw}{cw}$$

$$\tan \theta = \frac{s}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

From (9),

$$y + c = \sqrt{c^2 + s^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Therefore, (Fig. 8) if PD be parallel to YO and equal to $y + c$, as shown, and EP $(= s)$ be the tangent at P, then ED will be equal to c and perpendicular to tangent EP.

From (12) it may easily be seen that as y is a variable and c a constant, the point D for any other ordinate will necessarily lie on the line OD. This line OD is called the directrix of the catenary.

Equation (8) can be written,

$$e^{\frac{x}{c}} = \frac{s + \sqrt{c^2 + s^2}}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Where e is equal to 2.718, the base of the natural logarithms.
Solving for s in (13)

$$s = \frac{c}{2}\left[e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

From (9) and (14), the value of y in terms of x can be found. Remembering that in the notation of hyperbolic functions

$$\frac{1}{2}\left[e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right] \equiv \cosh \frac{x}{c}$$

and

$$\frac{1}{2}\left[e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right] \equiv \sinh \frac{x}{c}$$

equation (14) may be written

$$s = c \sinh \frac{x}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

and equation (9)

$$y = \sqrt{s^2 + c^2} - c = c\sqrt{\sinh^2 \frac{x}{c} + 1} - c$$

$$= c \cosh \frac{x}{c} - c$$

$$= \frac{c}{2}\left[e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right] - c \quad \ldots\ldots\ldots (16)$$

We know how to determine the tension at the lowest point of the wire (see equations (2) and (10)). It will now be shown how the tension at the highest point (at the insulator), or at any other point of the wire, can be determined.

Equation (3) may be written,

$$T = \frac{ws}{\sin \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

From Fig 8,

$$\sin \theta = \frac{s}{y + c}$$

Therefore

$$T = \frac{ws}{\sin \theta} = w(y + c) \quad \ldots\ldots\ldots\ldots\ldots (18)$$

This last equation shows that the tension at any point is equal to the weight of a portion of the wire whose length is equal to the ordinate of the point plus c.

If we call f the maximum value of y, i. e., the deflection of the wire, then from (2) and (18)

$$T = H + fw \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Remembering that $y_{max} = f$, equation (18) may be written

$$T = fw + cw \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Therefore

$$c = \frac{T - fw}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

Equating (10) and 21*a*)

$$\frac{s^2 - f^2}{2f} = \frac{T - fw}{w}$$

Therefore

$$T = \frac{s^2 w + f^2 w}{2f} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

Solving for f

$$f = \frac{T \pm \sqrt{T^2 - s^2 w^2}}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Equation (22) may be written

$$T = fw \pm \sqrt{T^2 - s^2 w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Equating (19) and (23)

$$H + fw = fw \pm \sqrt{T^2 - s^2 w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

and

$$H = \pm \sqrt{T^2 - s^2 w^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

This last equation shows that the tension T at the insulator, the tension H at the lowest point of the conductor, and the weight W of half the span of wire can be represented as shown in Fig. 5.

(The $\pm$ sign in equation (25) may be taken to show that when considering the right or left-hand half of the span of wire as free and in equilibrium, the horizontal tension H which should be put in

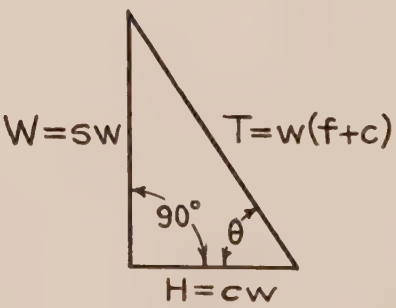

Fig. 9

(as shown in Fig. 9), would point in one or in the other direction.) Equating equations (2) and (25)

$$c\,w = \sqrt{T^2 - s^2 w^2}$$

whence

$$c = \frac{\sqrt{T^2 - s^2 w^2}}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

For all ordinary spans, the tension H at the lowest point of the wire is very large compared with the weight w per unit length. Hence from equation (2) the length c is large compared with x, and the ratio $\frac{x}{c}$ is therefore small. This fact permits the use of certain simplifying approximations that may be used if the span is not too long. In the following discussion, we shall first develop the approximate equations and show their application by a numerical example. We shall then take up the case of a very long span and show that for such a span the errors introduced by the approximation are too large to be neglected and that the exact analysis should be employed.

13. *Approximate or Parabola Method.*—Developing the functions $e^{\frac{x}{c}}$ and $e^{-\frac{x}{c}}$ by the aid of Maclaurin's formula,

$$e^{\frac{x}{c}} = 1 + \frac{x}{c} + \frac{x^2}{\underline{|2}c^2} + \frac{x^3}{\underline{|3}c^3} + \frac{x^4}{\underline{|4}c^4} + \text{etc.},$$

$$\text{and} \quad e^{-\frac{x}{c}} = 1 - \frac{x}{c} + \frac{x^2}{\underline{|2}c^2} - \frac{x^3}{\underline{|3}c^3} + \frac{x^4}{\underline{|4}c^4} - \text{etc.},$$

$$(\text{where } \underline{|2} = 1 \times 2, \underline{|3} = 1 \times 2 \times 3, \text{ etc}).,$$

Equation (14) therefore can be made to assume the form,

$$s = x + \frac{x^3}{\underline{|3}c^2} + \frac{x^5}{\underline{|5}c^4} + \frac{x^7}{\underline{|7}c^6} + \text{etc.} \quad \ldots\ldots\ldots\ldots (27)$$

In a similar manner, making $y_{\max} = f$, equation (16) may be changed to the form

$$f = \frac{x^2}{\underline{|2}c} + \frac{x^4}{\underline{|4}c^3} + \frac{x^6}{\underline{|6}c^5} + \text{etc.} \quad \ldots\ldots\ldots (28)$$

Since is small, the higher powers may be neglected for approxi-

mate analysis. If $l = 2x$ and $L = 2s$ (Fig. 10), equations (27) and

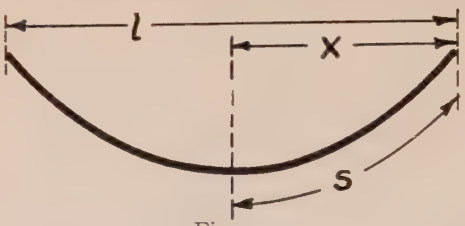

Fig. 10

(28) then become, respectively,

$$L = l + \frac{l^3}{24\,c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

$$f = \frac{l^2}{8c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The effect of this approximation is to substitute a parabola for the true catenary.

Solving for c in equation (30) and substituting (29)

$$L = l + \frac{8f^2}{3l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Combining (2) and (30)

$$f = \frac{l^2 w}{8H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

and

$$H = \frac{l^2 w}{8f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

Equation (32) gives the deflection f at the minimum temperature (say, — 20° F.) when w is made equal to the resultant force produced by the weight of wire plus the ice-covering (assumed to be ½in. thick) acting downward, and the maximum wind pressure (18 lb. per sq. ft.) acting normal to the combined weight.

Formula (31) gives the length of the wire at the minimum temperature (— 20° F). When the temperature rises to, say, 110° F., the equation for the length, L_1, of the wire assumes the form,

$$L_1 = l + \frac{8f^2}{3l} + \alpha\, Lt - \frac{L(H - H_1)}{AE} \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

where

l = length of span;
f = sag at minimum temperature (—20° F.);
a = coefficient of linear expansion per degree F.;
L = length of wire at minimum temperature;
t = difference between maximum and minimum temperature.
H = tension at lowest point of wire at minimum temperature;
H_1 = tension at lowest point of wire at maximum temperature;
A = cross-sectional area of wire;
E = modulus of elasticity of wire.

In formula (34) the term aLT represents the expansion of the wire due to rise of temperature and the term $L(H - H_1) \div A\,E$ represents the corresponding contraction produced by the decrease in tension due to the higher temperature.

Equation (31) may be written

$$f^2 = \frac{3}{8} l\ (L - l) \ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

In a similar manner, if f_1 is the deflection at maximum temperature and L_1 the corresponding length of the conductor

$$f_1^2 = \frac{3}{8} l\ (L_1 - l) \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

Substituting the value of L_1 given in (34), equation (36) becomes

$$f_1^2 = f^2 + \frac{3}{8} \frac{l\,L}{} \left(a\,t - \frac{H - H_1}{AE} \right) \quad \ldots\ldots (37)$$

Equation (31) may be put in the form,

$$L_1 = l + \frac{8 f_1^2}{3 l} \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Dividing (37) by (35) and solving for f_1,

$$f_1 = f \sqrt{\frac{L\ (1 + a\,t - \dfrac{H - H_1}{AE}) - l}{L - l}} \ldots\ldots\ldots (39)$$

In (39) there are two unknown quantities, f_1 and H_1. Therefore, the value of H_1 must be found in order to solve for f_1.

From (33)

$$H = \frac{l^2 w}{8 f} \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

and $$H_1 = \frac{l^2 w_1}{8 f_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

In (41) w_1 is used instead of w, because for temperatures above freezing point the ice load disappears. If we want to know both the maximum tension and maximum sag at any higher temperature, w_1 is, of course, the resultant, per unit length, of the weight of the conductor and the maximum wind pressure acting normal to the weight,

Combining (41) and (40), and solving for H_1

$$H_1 = \frac{f H w_1}{f_1 w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Substituting this value of H_1 in equation (37)

$$f_1^2 = f^2 + \frac{3 l L}{8}\left(\alpha T - \frac{H}{AE} + \frac{f H w_1}{AE f_1 w}\right) \quad \ldots\ldots\ldots\ldots (43)$$

Multiplying through by f_1, collecting terms and transposing

$$f_1^3 - f_1\left[f^2 + \frac{3 l L}{8}\left(\alpha T - \frac{H}{AE}\right)\right] - \frac{3 l L f H w_1}{8 A E w} = 0 \quad \ldots\ldots\ldots\ldots (44)$$

Equation (44) has only one unknown quantity, f_1, and may be written,

$$f_1^3 + 3 M f_1 + N = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

V. Practical Use of Foregoing Formulas.

To show how the preceding formulas can be used in solving practical problems, a numerical example will be given. Several assumptions will have of necessity to be made regarding temperature range, wind pressure, ice coating, etc. The minimum temperature T_2 will be assumed to be —20° F. and the maximum temperature T_1 equal to 110° F. Therefore, the temperature range will be assumed to be 130° F. A ½-in. coat of ice is a conservative figure, making the total diameter 1 in. plus the diameter of the wire or cable. The wind pressure for the ice-covered wire will be assumed to be 18 lb. per sq. ft. (counting total diameter into length as the exposed area). The modulus of elasticity, E*, of hard-drawn copper strands will be taken as 16,000,000 lb. per sq. in. The coefficient of expansion, α, per degree Fahrenheit is 0.00000956.

*Mr. F. O. Blackwell has shown that this value of E may be as low as 12 000 000 lb. per sq. in.

As an example, a strand having the following constants will be considered:

Strand = 37 hard-drawn copper wires about No. 10 B & S;
Diameter = 0.679 in.;
Area = 350,000 c.m. or 0.275 sq. in.;
Weight = 1.07 lb. per lin. ft.;
Elastic limit = 9640 lb. or 35,000 lb. per sq. in.

Taking the weight of ice as 0.0332 lb. per cu. in., the weight per ft. of the ½-in. coat of ice on the strand would be

$$0.0332 \times 12 \times \frac{\pi}{4}\left[(1.679)^2 - (0.679)^2\right] = 0.73 \text{ lb.}$$

Therefore, the total weight of and the wind pressure on the strand with its ice covering would be, respectively,

$$1.07 + 0.73 = 1.8 \text{ lb. per foot} \quad \ldots\ldots\ldots\ldots\ldots (46)$$

and

$$\frac{1.679 \times 12 \times 18}{144} = 2.52 \text{ lb.} \quad \ldots\ldots (47)$$

But (46) and (47) act at right angles to each other, and, therefore, the resultant force acting on the strand is,

$$w = \sqrt{(1.8)^2 + (2.52)^2} = 3.1 \text{ lb. per lin. ft.}$$

The permissible strain in the strand, if taken at two-thirds the elastic limit, would be

$$\frac{2}{3} \times 9640 = 6430 \text{ lb.}$$

Tabulating the data given above, together with the quantities to be determined,

l = length of span = 660 ft.
f = sag at minimum temperature (—20° F.).
f_1 = sag at higher temperature.
H = tension at lowest point of wire at minimum temperature = 6430 lb.
H_1 = tension at lowest point of wire at higher temperature.
w = resultant weight = 3.1 lb. per ft.
L = length of wire in span at minimum temperature.
L_1 = length of wire at higher temperature.
T = temperature range (=130° F.).
a = coefficient of linear expansion for copper = 0.00000956.

From equation (32), we find the sag at minimum temperature to be

$$f = \frac{l^2 w}{8H} = \frac{(660)^2 \times 3.1}{8 \times 6430} = 26.24 \text{ ft.}$$

From (31), the length of the wire at minimum temperature is

$$L = l + \frac{8f^2}{3l} = 660 + \frac{8(26.24)^2}{3 \times 660} = 662.78 \text{ ft.}$$

Before attempting to calculate either the deflection or the length of the cable at the maximum temperature (assumed to be 110°F.) the force acting on this conductor when the ice load disappears must be determined. The weight of the cable is 1.07 lb. per linear ft. The wind pressure acting on it (assuming 18 lb. per sq. ft.), would be

$$\frac{0.679 \times 12 \times 18}{144} = 1.02 \text{ lb. per lin. ft.}$$

Therefore, as these two forces act at 90° to each other, the resultant force is

$$w_1 = \sqrt{(1.07)^2 + (1.02)^2} = 1.48 \text{ lb. per ft.}$$

We are now ready to calculate the deflection of the strand when the thermometer stands at 110° F., and the wind is blowing at such a velocity as to produce on the strand a pressure of 18 lb. per sq. ft.

From (44) and (45), making the required substitutions, the values of M and N are found to be

$$M = -217.5 \quad \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

and

$$N = -3005 \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

Substituting the numerical values given in (48) and (49), equation (45) assumes the form

$$f_1^3 - 652.5\ f_1 = 3005 \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

The above equation solved either by trial or by Horner's method, gives the value of the maximum deflection as

$$f_1 = 27.62 \text{ feet} \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

From (41), the tension in the strand when both temperature and wind pressure reach a maximum value is

$$H_1 = \frac{l^2 w_1}{8f_1} = \frac{(660)^2 \times 1.48}{8 \times 27.62} = 2920 \text{ lb.} \ldots\ldots\ldots\ldots (52)$$

That is, the decrease in tension due to rise in temperature is

$$H - H_1 = 6430 - 2920 = 3510 \text{ lb.} \ldots\ldots\ldots\ldots (53)$$

As a check on the above value, the length of the strand at maximum temperature will be found following two quite different methods. From equation (38)

$$L_1 = l + \frac{8f_1^2}{3l} = 660 + \frac{8(27.62)^2}{3 \times 660} = 663.08 \text{ ft.} \quad \ldots\ldots (54)$$

From (34) and (54)

$$L_1 = l + \frac{8f^2}{3l} + \alpha LT - \frac{L(H - H_1)}{AE}$$

$$= 660 + 2.78 + 0.823 - 0.529.$$

$$= 663.074 \text{ ft.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Comparing the lengths as given by (54) and (55), we can readily see that they are practically the same. This close agreement very clearly shows that the value of the maximum deflection given in (52) is the correct one.

VI. Catenary Method

The question has often been asked,—are engineers justified in assuming the curve subtended by a span of wire to be a parabola instead of a true catenary? The writer will endeavor to answer this question, giving a numerical illustration. It is not necessary to mention short spans and small sags because in this case, as it is well known, for all practical purposes, the results obtained by one or the other method are identical. To illustrate this point better, the same 350,000 c.m. copper strand will be discussed. The same constants will be used; but the curve will be regarded as a true catenary.

The tension H at the lowest point of the span can be found using equation (19). Thus

$$T = H + fw = 6430 = H + 26.24 \times 3.1;$$

Therefore,

$$H = 6430 - 81.34 = 6348.66 \text{ lb.}$$

From equation (2)

$$c = \frac{H}{w} = \frac{6348.66}{3.1} = 2048$$

The approximate value of f (=26.24 ft.; see Table 4) has been used above, because in the present case it can easily be proved that 10

per cent error in the value of the deflection would cause but 0.15 per cent error in the value of the tension constant c. This assumption would be justified even if considering the longest span and the greatest sag in existence. In the present case, the error introduced is only 1.45 per cent (see Table 4) instead of 10 per cent, and, therefore, the error in the value of c will be less than 0.02 per cent.

From (14)

$$s = \frac{c}{2}\left[e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

Let, as before, $l = 2x$ and $L = 2s$.

Then

$$\frac{x}{c} = \frac{l}{2c} = \frac{660}{2 \times 2048} = 0.1611.$$

Making the required substitutions, equation (56) assumes the form

$$L = c\left[e^{\frac{l}{2c}} - e^{-\frac{l}{2c}}\right] = 2048\left[e^{0.1611} - e^{-0.1611}\right]$$

$$= 2048\left[1.1748 - 0.8512\right] = 662.74 \text{ ft.}$$

The values of e in the above were found by interpolation, using Table 5.

The deflection of the strand at $-20°$F can be determined as shown below.

$$f = \sqrt{s^2 + c^2} - c = \sqrt{(2048)^2 + (311.37)^2} - 2048$$

$$= 26.62 \text{ ft.}$$

Also

$$f = \frac{c}{2}\left[e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right] - c$$

$$= 1024\left[1.1748 + 0.8512\right] - 2048 = 26.62 \text{ ft.}$$

(For values of e see Table 5.)

Knowing very well that the equations for the catenary are most unwieldly, the writer had almost given up hope of deducing accurate equations for length, sag, etc., which, considering the curve as a catenary, would take into account wind pressure, ice load, temperature changes and corresponding elongation and contraction, etc. After

repeated failure, he succeeded in deducing an equation, the derivation of which will be briefly given below.

Equation (34) may be written,

$$s_1 = s + a s\tau - \frac{s(T - T_1)}{AE} \quad \ldots\ldots\ldots\ldots\ldots (57)$$

where

s_1 = half length of wire at higher temperature.
s = half length of wire at minimum temperature.
τ = temperature range.
T_1 = tension at insulator at higher temperature.

Combining (20) and (16), solving for T_1

$$T_1 = \frac{w_1 c_1}{2}\left[e^{\frac{x}{c_1}} + e^{-\frac{x}{c_1}}\right] \quad \ldots\ldots\ldots\ldots\ldots (58)$$

Substituting this value in (58)

$$s_1 = s + as\tau - \frac{s\left[T - \frac{w_1 c_1}{2}\left(e^{\frac{x}{c_1}} + e^{-\frac{x}{c_1}}\right)\right]}{AE} \quad \ldots\ldots (59)$$

From (14)

$$s_1 = \frac{c_1}{2}\left[e^{\frac{x}{c_1}} - e^{-\frac{x}{c_1}}\right] \quad \ldots\ldots\ldots\ldots\ldots (60)$$

From (59) and (60)

$$s\left(1 + a\tau - \frac{T}{AE}\right) = \frac{c_1}{2}\left[\left(1 - \frac{sw_1}{AE}\right)e^{\frac{x}{c_1}} - \left(1 + \frac{sw_1}{AE}\right)e^{-\frac{x}{c_1}}\right] \quad \ldots\ldots (61)$$

All the equations are known except c_1, and by trial the exact value of this constant can be determined.

To show the application of the above formulas for the catenary, a numerical example will be given. As before, the same copper strand, the same span, etc., will be considered.

Substituting the numerical values already given, formula (61) assumes the form

$$331.71 = \frac{c_1}{2}\left[e^{\frac{330}{c_1}} \times 0.9999 - 1.000112\, e^{-\frac{330}{c_1}}\right] \quad \ldots\ldots\ldots (62)$$

Therefore

$$c_1 = 1872 \quad \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

An almost correct value of c_1 could have been obtained using (27) in the following convenient form

$$s\left(1+\alpha\tau-\frac{T}{AE}\right)=x+\frac{x^3}{6c_1^2}$$

$$=331.71=330-\frac{(330)^3}{6c_1^2}$$

Therefore

$$c_1=1871.1.$$

As seen, the two values for c_1 are almost identical. This holds even for very long spans.

Subsituting in (60) the value of c_1 given by the exact formula, the length of the cable is found to be

$$L_1=2s_1=c_1\left[e^{\frac{x}{c_1}}-e^{-\frac{x}{c_1}}\right]$$

$$=1872\ (1.1928-0.8384)=663.44\ \text{ft.}$$

From (12), the deflection is found to be

$$f_1=\sqrt{c_1^2+s_1^2}-c_1$$

$$=\sqrt{(1872)^2+(331.72)^2}-1872=29.17\ \text{ft.}$$

Another way, perhaps more convenient, is to obtain the deflection from (16), as follows:

$$f_1=\frac{c_1}{2}\left[e^{\frac{x}{c_1}}+e^{-\frac{x}{c_1}}\right]-c_1$$

$$=\frac{1872}{2}(1.1928+0.8384)-1872=29.2\ \text{ft.}$$

From an inspection of Table 4 the conclusion is reached that for very long spans and great sags, the deflections and tensions obtained by the parabola method are by no means as accurate as they should be. The present long span and the much longer span of the future require something better. The writer believes that the catenary method suggested is simple and accurate, and could easily be used in practice.

TABLE 4

660-FT. SPAN. SAGS, TENSIONS, AND LENGTHS OF 350,000 C. M. HARD-DRAWN COPPER STRAND.

Catenary	Temperature,—20°F. Parabola	Per cent Error	Catenary	Temperature, 110°F. Parabola	Per cent Error
		SAG IN FEET			
C	P	(C-P)÷P	C	P	(C-P)÷P
26.62	26.24	1.45	29.20	27.62	5.72
		LENGTH IN FEET			
C	P	(P-C)÷C	C	P	(C-P)÷P
662.74	662.78	0.006	663.44	663.08	0.05
		TENSION IN POUNDS			
C	P	(C-P)÷P	C	P	(P-C)÷C
6430	6430	0	2814	2920	3.77

TABLE 5

HYPERBOLIC FUNCTIONS

$\frac{x}{c}$	$e^{\frac{x}{c}}$	$e^{-\frac{x}{c}}$	$\text{Sinh}\ \frac{x}{c}$	$\text{Cosh}\ \frac{x}{c}$
0.00	1.0000	1.0000	0.0000	1.0000
.01	1.0100	0.9900	.0100	1.0000
.02	1.0202	.9802	.0200	1.0002
.03	1.0305	.9704	.0300	1.0004
.04	1.0408—	.9608	.0400	1.0008
.05	1.0513	.9512	.0500	1.0013
.06	1.0618	.9418	.0600	1.0018
.07	1.0725	.9324	.0701	1.0025
.08	1.0833	.9231	.0801	1.0032
.09	1.0942	.9139	.0901	1.0041
.10	1.1052	.9048	.1002	1.0050
.11	1.1163	.9858	.1102	1.0061
.12	1.1275	.8869	.1203	1.0072
.13	1.1388	.8781	.1304	1.0085
.14	1.1503	.8694	.1405	1.0098
.15	1.1618	.8607	.1506	1.0113
.16	1.1735	.8521	.1607	1.0128
.17	1.1853	.8437	.1708	1.0145
.18	1.1972	.8353	.1810	1.0162
.19	1.2092	.8270	.1911	1.0181
.20	1.2214	.8187	.2013	1.0201
.21	1.2337	.8106	.2115	1.0221
.22	1.2461	.8025	.2218	1.0243
.24	1.2712	.7866	.2423	1.0289

LIST OF PUBLICATIONS

ALL BULLETINS, THE TITLES OF WHICH ARE NOT STARRED, WILL BE SENT FREE UPON APPLICATION.

**Bulletin No.* 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

**Circular No.* 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*

**Bulletin No.* 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

**Circular No.* 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

**Circular No.* 3. Fuel Tests with Illinois Coal. (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907,) by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

**Bulletin No.* 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

**Bulletin No.* 4. Tests of Reinforced Concrete Beams, Series of 1905. by Arthur N. Talbot. 1906. *Forty-five cents.*

**Bulletin No.* 5. Resistance of Tubes to Collapse, by Albert P. Carman. 1906. *Fifteen cents.*

**Bulletin No.* 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *Thirty-five cents.*

**Bulletin No.* 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr and Henry B. Dirks. 1906. *Thirty-five cents.*

**Bulletin No.* 8. Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

**Bulletin No.* 9. An Extension of the Dewey Decimal System of Classification Applied to the Enginering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. *None available.*

**Bulletin No.* 10. Tests of Concrete and Reinforced Concerte Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No.* 11. The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *Fifteen cents.*

**Bulletin No.* 12. Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No.* 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

**Bulletin No.* 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

**Bulletin No.* 15. How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

**Bulletin No.* 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

**Bulletin No.* 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *Twenty cents.*

**Bulletin No.* 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

**Bulletin No.* 19. Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *Twenty-five cents.*

**Bulletin No.* 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

**Bulletin No.* 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *Thirty-five cents.*

Bulletin No. 23. Voids, Settlement and Weight of Crushed Stone, by Ira O. Baker, 1908. *Fifteen cents.*

Bulletin No. 24. The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Thirty cents.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

---

*Out of print; price attached.

Bulletin No. 26. High Steam-Pressures in Locomotive Service. A Review of a Report to the Carnegie Institute of Washington, by W. F. M. Goss. 1908. *Twenty-five cents.*

Bulletin No. 27. Tests of Brick Columns and Terra Cotta Block Columns, by Arthur N. Talbot and Duff A. Abrams. 1909. *Thirty cents.*

Bulletin No. 28. A Test of Three Large Reinforced Concrete Beams, by Arthur N. Talbot. 1909. *Fifteen cents.*

Bulletin No. 29. Tests of Reinforced Concrete Beams: Resistance to Web Stresset, Series of 1907 and 1908, by Arthur N. Talbot. 1909. *Forty-five cents.*

Bulletin No. 30. On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Twenty-five cents.*

Bulletin No. 31. Fuel Tests with House-Heating Boilers, by J. M. Snodgrass. 1909. *Fifty-five cents.*

Bulletin No. 32. The Occluded Gases in Coal, by S. W. Parr and Perry Barker. 1909. *Fifteen cents.*

Bulletin No. 33. Tests of Tungsten Lamps, by T. H. Amrine and A. Guell. 1909. *Twenty cents.*

Bulletin No. 34. Tests of Two Types of Tile Roof Furnaces under a Water-tube Boiler, by J. M. Snodgrass. 1909. *Fifteen cents.*

**Bulletin No.* 35. A Study of Base and Bearing Plates for Columns and Beams, by N. Clifford Ricker. 1909. *Twenty cents.*

Bulletin No. 36. The Thermal Conductivity of Fire-Clay at High Temperatures, by J. K. Clement and W. L. Egy. 1909. *Twenty cents.*

**Bulletin No.* 37. Unit Coal and the Composition of Coal Ash, by S. W. Parr and W. F. Wheeler. 1909. *Thirty-five cents.*

Bulletin No. 38. The Weathering of Coal, by S. W. Parr and W. F. Wheeler. 1909. *Twenty-five cents.*

Bulletin No. 39. Tests of Washed Grades of Illinois Coal, by C. S. McGovney. 1909. *Seventy-five cents.*

Bulletin No. 40. A Study in Heat Transmission, by J. K. Clement and C. M. Garland. 1910. *Ten cents.*

Bulletin No. 41. Tests of Timber Beams, by Arthur N. Talbot. 1910. *Twenty cents.*

Bulletin No. 42. The Effect of Keyways on the Strength of Shafts, by Herbert F. Moore. 1910. *Ten cents.*

Bulletin No. 43. Freight Train Resistance, by E. C. Schmidt. 1910. *Eighty cents.*

Bulletin No. 44. An Investigation of Built-up Columns under Load, by Arthur N. Talbot and Herbert F. Moore. 1911. *Thirty-five cents.*

Bulletin No. 45. The Strength of Oxyacetylene Welds in Steel, by Herbert L. Whittemore. 1911. *Thirty-five cents.*

Bulletin No. 46. The Spontaneous Combustion of Coal, by S. W. Parr and F. W. Kressmann. 1911. *Forty-five cents.*

Bulletin No. 47. Magnetic Properties of Heusler Alloys, by Edward B. Stephenson. 1911. *Twenty-five cents.*

Bulletin No. 48. Resistance to Flow through Locomotive Water Columns, by Arthur N. Talbot and Melvin L. Enger. 1911. *Forty cents.*

Bulletin No. 49. Tests of Nickel Steel Riveted Joints, by Arthur N. Talbot and Herbert F. Moore. 1911. *Thirty cents.*

Bulletin No. 50. Tests of a Suction Gas Producer, by C. M. Garland and A. P. Kratz. 1911. *Fifty cents.*

Bulletin No. 51. Street Lighting, by J. M. Bryant and H. G. Hake. 1911. *Thirty-five cents.*

Bulletin No. 52. An Investigation of the Strength of Rolled Zinc, by Herbert F. Moore. 1912. *Fifteen cents.*

Bulletin No. 53. Inductance of Coils, by Morgan Brooks and H. M. Turner. 1912. *Forty cents.*

Bulletin No. 54. Mechanical Stresses in Transmission Lines, by A. Guell. 1912. *Twenty cents.*

ANNOUNCEMENT CONCERNING A MODIFICATION IN THE RULES GOVERNING THE DISTRIBUTION OF BULLETINS

The Board of Trustees of the University of Illinois voted, December 3, 1910, that a price should be affixed to certain University publications, among them being the Bulletin of the Engineering Experiment Station.

This action, so far as it concerns the bulletins of the Engineering Experiment Station, has for its purpose a threefold object:

(1) To provide a greater degree of control in the distribution of bulletins.

(2) To make possible the establishment and maintenance of a trade circulation through the regular publishing houses.

(3) To regulate the distribution of the reserve or "out-of-print" supply.

IT IS NOT INTENDED THAT THIS ACTION SHALL OPERATE IN ANY WAY TO ABRIDGE THE PRIVILEGES OF THOSE WHO HAVE HITHERTO RECEIVED THE BULLETINS OF THE STATION GRATUITOUSLY, OR TO PREVENT REASONABLE EXTENSIONS OF THE EXISTING MAILING LISTS.

Practice under the new procedure will be as follows:

(1) All bulletins, hereafter issued, will have a price printed upon the cover, together with the name of the London sales agent.

(2) EACH BULLETIN, HOWEVER, WILL STILL BE SUBJECT TO A FREE INITIAL DISTRIBUTION, AS HERETOFORE, ON THE BASIS OF EXISTING MAILING LISTS.

It will also be placed on sale with authorized agencies, both in this country and abroad.

(3) THERE WILL BE A LIMITED NUMBER OF COPIES AVAILABLE FOR FREE DISTRIBUTION, UPON REQUEST, AFTER THE INITIAL DISTRIBUTION.

(4) As the supply of each bulletin approaches exhaustion, it will be placed upon a reserve or "out-of-print" list. THE EFFECT OF THIS ACTION WILL BE TO WITHDRAW SUCH BULLETINS FROM FREE DISTRIBUTION. Bulletins withdrawn from free distribution will be available to any applicant upon payment of the assigned price.

(5) A discount from the published price will be allowed to news agents regularly handling the bulletins.

W. F. M. GOSS,
Director.